Name: Feedback

SA402 · Dynamic and Stochastic Models

Fall 2023 · Uhan

Exam 2 – 11/1/2023

Instructions

- You have 50 minutes to complete this exam.
- You may use your plebe-issue TI-36X Pro calculator.
- You may not use any other materials.
- **No collaboration allowed.** All work must be your own.
- **Show all your work.** To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.
- **Do not discuss the contents of this exam with any midshipmen until it is returned to you.**

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
1d	1	
2a	1	
2b	1	
3a	1	
3b	1	
4	1	
5	1	
Total		/ 100

Problem 0. Copy and sign the honor statement below. This exam will not be graded without a signed honor statement.

The Naval Service I am a part of is bound by honor and integrity. I will not compromise our values by giving or receiving unauthorized help on this exam.

Signature:

Problem 1. You have been hired as an operations research analyst at the Simplexville Hospital. According to your predecessor's notes, vehicles arrive at the entrance of the parking garage according to a stationary Poisson process at a rate of 27 per hour between 6:00 and 21:00.

a. What is the probability that 120 or fewer vehicles arrive at or before 12:00, given that exactly 70 vehicles arrive between 6:00 and 10:00?

See Lesson 9 and the Lesson 9 Exercises for a number of similar examples.

b. What is the expected number of vehicles by the end of the day (6:00 - 21:00), given that exactly 300 vehicles arrive between 6:00 and 15:00?

See the Example 4 in Lesson 9 and Problem 1c in the Review Problems for Exam 2 for similar examples.

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c. What is the expected time of the 80th vehicle arrival?

See Example 5 in Lesson 9 and Problem 4d in the Lesson 9 Exercises for similar examples.

d. Suppose it is 13:00. What is the probability that the next vehicle arrives within the next minute (1/60 hour)?

See Problem 2c in the Lesson 9 Exercises and Problem 1a in the Review Problems for Exam 2 for similar examples.

Problem 2. The main lobby at the Simplexville Hospital is open from 6:00 to 21:00. Again, according to your predecessor, employees arrive at the main lobby at a rate of 9 per hour, and visitors arrive at a rate of 17 per hour. Model the arrivals at both entrances as independent, stationary Poisson processes.

a. What is the probability that the 40th person (either employee or visitor) arrives at the lobby at or before 8:00?

See Example 4 in Lesson 10 for a similar example.

b. What is the probability that 10 or more employees arrive at the lobby during <u>each</u> of 2 consecutive hours? (For example, 10 or more employees arrive between 12:00 and 13:00, and then 10 or more employees arrive between 13:00 and 14:00.)

See Problem 5d in the Lesson 10 Exercises and Problem 1e in the Review Problems for Exam 2 for similar examples.

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Problem 3. After working at the Simplexville Hospital for a few days, you start to suspect that your predecessor was wrong about the parking garage, so you collect your own data. You find that the vehicles actually arrive at the entrance of the parking garage according to a nonstationary Poisson process with the following integrated rate function:

$$\Lambda(\tau) = \begin{cases} 40\tau & \text{if } 0 \leq \tau < 3 \\ 10\tau + 90 & \text{if } 3 \leq \tau < 9 \\ 30\tau - 90 & \text{if } 9 \leq \tau < 12 \\ 5\tau + 210 & \text{if } 12 \leq \tau \leq 15 \end{cases}$$

where $\tau = 0$ corresponds to 6:00 and $\tau = 15$ corresponds to 21:00.

a. What is the expected number of arrivals between 12:00 and 16:00?

See Examples 1b and 2c in Lesson 11, as well as Problem 2b from the Lesson 11 Exercises for similar examples.

b. What is the probability that 47 or more vehicles arrive between 8:00 and 10:00?

See Problems 2c and 3a from the Lesson 11 Exercises for similar examples.

Problem 4. After the parking garage debacle, you start to suspect that your predecessor was wrong about the lobby as well. After collecting your own data, you find that people (employees and visitors) actually arrive at the lobby according to a nonstationary Poisson process with the arrival rate function below:

$$\lambda(\tau) = \begin{cases} 32 & \text{if } 0 \leq \tau < 4 \\ 8 & \text{if } 4 \leq \tau < 11 \\ 12 & \text{if } 11 \leq \tau \leq 15 \end{cases}$$

where $\tau = 0$ corresponds to 6:00 and $\tau = 15$ corresponds to 21:00.

What is the integrated rate function for this nonstationary Poisson process?

See Example 2a in Lesson 11, as well as Problems 1a and 2a in the Lesson 11 Exercises for similar examples.

Problem 5. You have now turned your attention to the cafeteria at the hospital. Your colleague believes that customers arrive at the entrance to the cafeteria according to a stationary Poisson process. Describe what assumptions need to be made about the customer arrivals in order for this to be true. (You do not need to assess whether these assumptions are realistic.)

Note that "stationary Poisson process" is not the same as "stationary increments". "Stationary Poisson process" refers to a Poisson process with a stationary or constant arrival rate, unlike a "nonstationary Poisson process".

See Problem 3 from the Review Problems for Exam 2 for a similar example. Keep your answers simple and precise.